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Superconductivity Driven by Chain Coupling and Electronic Correlations

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Abstract. – We present an analysis of a system of weakly coupled Hubbard chains based on combining an exact study of spectral functions of the uncoupled chain system with a renormalization group method for the coupled chains. For low values of the onsite repulsion U and of the doping δ, the leading instability is towards a superconducting state. The process includes excited states above a small correlation pseudogap. Similar features appear in extended Hubbard models in the vicinity of commensurate fillings. Our theoretical predictions are consistent with the phase diagram observed in the $(TMTTF)_2X$ and $(TMTSF)_2X$ series of organic compounds.

Different classes of materials such as (i) high-T_c superconductors, (ii) quasi-one dimensional (1D) organic conductors, and (iii) heavy-Fermion compounds show superconductivity (SC) phases whose physics is far from being well understood [1]. All these materials have as common feature the existence of non-Fermi liquid (NFL) metallic phases. In addition, in classes (i) and (ii) electronic transport is quite anisotropic, space dimensionality being expected to play an important role in the observed exotic spectral properties. On the other hand, SC phases were not observed in incommensurate quasi-1D materials [2, 3].

The Hubbard model has attracted much interest as the simplest model for the description of the electronic properties of these *low-dimensional* materials. While the 1D Hubbard model has no SC phases for on-site repulsion $U > 0$ [4], there has been evidence that the two-dimensional (2D) Hubbard model has a d-wave pairing instability [5]. In addition, the undoped 1D Hubbard model is the prototype used in the study of the confinement problem [6]. Therefore, the study of the interplay of the crossover from one- to higher-dimensional systems with the electronic correlations is of great importance for the understanding of the exotic physics detected in the above materials [1]. An interesting toy model for that important problem is a system of $N_{ch} \rightarrow \infty$ weakly coupled Hubbard chains, each chain having $N_a \rightarrow \infty$ sites and the hopping integrals such that $t_{\parallel} > t_{\perp}$. Here $t_{\parallel}$ and $t_{\perp}$ correspond to electron transfer along each chain and between first-neighbor chains, respectively. Importantly, we find below

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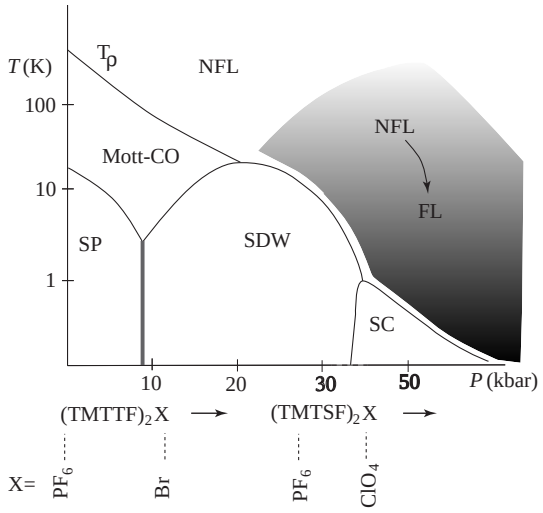

Fig. 1 – T, P Phase diagram of the $(TMTTF)_2X$ and $(TMTSF)_2X$ series of compounds.

that for electronic doping concentrations $\delta \neq 0$, where $\delta \equiv (1-n)$ and n is the electronic density, the singlet SC spectral functions of the one-chain problem show singularities for energy values just above a correlation pseudogap, E_{uhb}, which equals the Mott Hubbard gap E_{MH} for $\delta \to 0$ [7]. Note that half filling for the coupled-chain system could correspond to a small effective doping δ for the single chains, due to charge fluctuations introduced by the inter-chain hopping [8]. Indeed, the key point is that for low values of $U/t_{\parallel}$ and of doping δ there is a regime where $E_{uhb} \approx |\delta|\, 2\pi t_{\parallel} + [8\sqrt{U t_{\parallel}}/\pi]\, e^{-2\pi t_{\parallel}/U}$ is small compared to $t_{\parallel}$ and one can choose $t_{\perp}$ such that $E_{uhb} < t_{\perp} < t_{\parallel}$. In such a regime the above finite-energy singularities of the one-chain problem give rise to relevant instabilities in the system of weakly coupled chains. Therefore, a complete study of the instabilities of the coupled chains having as starting point the properties of the one-chain problem must take into account both the low- and finite-energy singularities in the one- and two-electron spectral functions of the 1D problem.

In this Letter we combine the *pseudofermion dynamical theory* (PDT) for these spectral functions [3, 9, 10] with a renormalization group (RG) analysis [4, 11, 12] to study the instabilities of the above system of weakly coupled Hubbard chains. For the $(TMTTF)_2X$ and $(TMTSF)_2X$ series of organic compounds, whose phase diagram is represented in Fig. 1, we identify E_{uhb} with the experimental *correlation gap* [13] of Fig. 3 of Ref. [8]. Above the energies/temperatures of the inter-chain hopping and electron-phonon interactions, the 1D electronic correlations are expected to dominate. Indeed, the 1D Hubbard model has a NFL metallic phase for $\delta \neq 0$ and a Mott-Hubbard insulator phase for $\delta = 0$, consistently with the occurrence of NFL and Mott-CO phases in the diagram of Fig. 1. This model was used successfully in the study of the photoemission spectrum of the incommensurate organic compound TTF-TCNQ [2, 3, 10, 14]. The description of the above quarter-filled compounds involves extended Hubbard chains. However, due to dimerization [13] the low-T problem can be mapped onto our toy model for half filling, with U being an effective interaction parameter whose value depends on the bare on-site and first-neighbor (V) repulsions. Importantly, for energy and/or T values just above the dimerization gap, this leads to a correct description also for the small-δ metallic phase, provided that in our spectral-function expressions δ is replaced by an effective doping $\bar{\delta} \equiv (1/2 - n)$ relative to quarter filling. (For instance, for en-

ergy/frequency $(\omega - E_{MH}) > 0$ small, the optical conductivity reads $\mathrm{Re}\,\sigma(\omega) \propto (\omega - E_{MH})^{1/2}$ for both the $\delta = 0$ 1D Hubbard model [15] and the $\bar{\delta} = 0$ spinless fermion model with first-neighbor repulsion V [16].) We include in the perturbative expansion the inter-chain hopping $t_\perp$. In this respect our method differs from alternative approaches where the expansion parameter used is the electron-electron interaction U [17]. The latter perturbation scheme on U misses the effects that we discuss here. The influence of the inter-chain hopping terms has been studied by an expansion in the inverse of the chain coordination number [18].

The single Hubbard chain in a chemical potential μ reads $\hat{H} = \hat{H}_{SO(4)} - \mu[N_a - N]$, where $\hat{H}_{SO(4)} = \hat{T} + U\sum_j[\hat{n}_{j,\uparrow} - 1/2][\hat{n}_{j,\downarrow} - 1/2]$, $\hat{T} = -t_\parallel \sum_{j,\sigma}[c^\dagger_{j\sigma}c_{j+1\sigma} + h.c.]$, $N = \sum_\sigma N_\sigma$, $N_\sigma = \sum_j n_{j,\sigma}$, $c^\dagger_{j\sigma}$ and $c_{j\sigma}$ are electron operators of spin projection σ at site $j = 1, ..., N_a$, and $n_{j,\sigma} = \langle c^\dagger_{j\sigma}c_{j\sigma}\rangle$. $\hat{H}_{SO(4)}$ commutes with the six generators of the η-spin and spin algebras [7, 9, 19] and hence the η-spin value η and $\eta_z = -\delta N_a/2$ are good quantum numbers. We consider the $0 < \delta < 1$ metallic and $\delta = 0$ insulator phases with zero magnetization for the initial ground state (GS). The electronic charge reads $-e$ and $2k_F = (1-\delta)\pi$. We study the spectral functions $B^{(l)}_\vartheta(k,\omega)$ where $l = \pm 1$ and $B^{(+1)}_\vartheta(k,\omega) = \sum_f |\langle f|\hat{\mathcal{O}}^\dagger_\vartheta(k)|GS\rangle|^2\,\delta(\omega - \omega_{f,0})$ and $B^{(-1)}_\vartheta(k,\omega) = \sum_{f'} |\langle f'|\hat{\mathcal{O}}_\vartheta(k)|GS\rangle|^2\,\delta(\omega + \omega_{f',0})$ for $\omega > 0$ and for $\omega < 0$, respectively, whose singularities we detect and characterize. The f and f' summations run over the possible excited states and $\omega_{f,0}$ and $\omega_{f',0}$ are the energies relative to the initial GS. We find that the more divergent singular features occur for one-electron $\vartheta = 1p$, transverse spin density wave (SDW) $\vartheta = sdw$, and on-site and extended singlet SC $\vartheta = oss$ and $\vartheta = ess$, respectively. The corresponding operators read $\hat{\mathcal{O}}_{1p}(k) \equiv c_{k,\sigma}$, $\hat{\mathcal{O}}_{sdw}(k) \equiv \sum_{k'} c^\dagger_{k+k',\downarrow}c_{k',\uparrow}$, $\hat{\mathcal{O}}_{oss}(k) \equiv \sum_{k'} c_{k-k',\uparrow}c_{k',\downarrow}$, and $\hat{\mathcal{O}}_{ess}(k) \equiv \sum_{k'} \cos(k')[c_{k-k',\uparrow}c_{k',\downarrow} - c_{k-k',\downarrow}c_{k',\uparrow}]$. Here $c^\dagger_{k,\sigma}$ and $c_{k,\sigma}$ are spin $\sigma = \uparrow, \downarrow$ electron operators of momentum k. For the above 1D model, electrons are related to *rotated electrons* by a unitary transformation and *rotated-electron double occupancy* D_r is a good quantum number for all values of the on-site repulsion U [7,9]. For $0 \leq \delta \leq 1$, $D_r = 0$ for the GS and the first and second upper Hubbard bands (UHBs) are spanned by $D_r = 1$ and 2 excited states, respectively [7, 9, 10], whose lower limits are at energy $D_r\,E_{uhb}$. The correlation pseudogap E_{uhb} is called E_u in Ref. [10] and is such that $E_{uhb} = 4t_\parallel \sin(\pi\delta/2)$ for $U/t_\parallel \rightarrow 0$, $E_{uhb} = U - 4t_\parallel\cos(\pi\delta)$ for $U >> t_\parallel$, $E_{uhb} = U + 4t_\parallel$ for $\delta \rightarrow 1$, and $E_{uhb} = E_{MH}$ for $\delta \rightarrow 0$.

Our study of the above spectral functions $B^{(l)}_\vartheta(k,\omega)$ takes into account all microscopic processes described by the exact PDT introduced in Ref. [9]. The excited states can be described in terms of occupancy configurations of holons, spinons, and c pseudofermions [9,10]. We denote the η spin (and spin) and η-spin (and spin) projection of these objects by s_c and σ_c (and s_s and σ_s), respectively. The holons (and spinons) have $s_c = 1/2$, $\sigma_c = \pm 1/2$, charge $\pm 2e$, and $s_s = 0$ ($s_s = 1/2$, $\sigma_s = \pm 1/2$, and no charge degrees of freedom). The c pseudofermions carry charge $-e$ and have no spin and η-spin degrees of freedom. We use the notation $\pm 1/2$ holons (and $\pm 1/2$ spinons) according to the values of σ_c (and σ_s). The rotated-electron double occupation D_r equals the number of $-1/2$ holons. In the vicinity of the singular spectral features the spectral functions can be expressed as a power-law expansion whose small parameter for a given value of the momentum is the energy deviation from the singular feature. For the one- and two-electron spectral functions only specific types of $\Delta D_r = 0$ and/or $\Delta D_r = 1$ processes contribute to the leading-order term of such a power-law expansion. (Here $\Delta D_r = D_r$ is the deviation from the ground-state value $D_r = 0$.) We call these processes *dominant processes*. All remaining processes give rise to higher-order power-law contributions, controlled by exponents larger than those studied in this paper. The evaluation of the weight distribution in the vicinity of the singular features of the above two-electron spectral functions proceeds

as for the one-electron case reported in Ref. [10]. The dominant processes involve changes in the occupancies of the c pseudofermions, two-holon $s_c = 0$ composite $c1$ pseudofermions, two-spinon $s_s = 0$ composite $s1$ pseudofermions, and localized and non-interacting $-1/2$ Yang holons, which are not part of composite pseudofermions [7,9]. There are no $-1/2$ Yang holons and $c1$ pseudofermions in the $D_r = 0$ GS. The $\alpha = c, s1, c1$ pseudofermions carry momentum $\bar{q} = q + Q_\alpha(q)/N_a$. Here q is the *bare-momentum* and $Q_\alpha(q)$ is the phase-shift functional defined in Ref. [9], whose expression involves the bare-momentum two-pseudofermion phase shifts $\Phi_{\alpha\,\alpha'}(q, q')$ in units of 2π, where α, $\alpha' = c, s1, c1$. (Such phase shifts are studied in Ref. [10].) The pseudofermions are related to the corresponding *pseudoparticles* of Ref. [7] by a unitary transformation [9]. The $c1$ energy dispersion $\epsilon_{c1}(q) = E_{uhb} + \epsilon^0_{c1}(q)$ such that $|\,q| \leq \delta\pi$ plays an important role in our study and is plotted in Ref. [7].

Nearly the whole Cooper-pair addition spectral weight is generated by processes such that $\Delta D_r = 0, 1, 2$. These processes generate three separated parts $B^{(+1)}_{\vartheta, D_r}(k, \omega)$ for the SC spectral functions, $B^{(+1)}_{\vartheta}(k, \omega) = \sum_{D_r=0,1,2} B^{(+1)}_{\vartheta, D_r}(k, \omega)$. At $\delta = 0$ only $D_r = 2$ excited states are allowed and thus $B^{(+1)}_{\vartheta}(k, \omega) = B^{(+1)}_{\vartheta, 2}(k, \omega)$ has no singular spectral features. For $\delta > 0$ and $U > 0$ the SC singular spectral features result from transitions to $D_r = 1$ excited states. The most divergent singularities in the SC functions occur for $\vartheta = oss$, ess in $B^{(+1)}_{\vartheta, 1}(k, \omega)$ and result from finite-energy transitions involving creation of a $c1$ pseudofermion and corresponding to ω just above the two $\iota' = \pm 1$ singular branch lines $E^{\iota'}_{ss}(k) = \epsilon_{c1}(q)$ where $q = [\iota'\, 2k_F - k]$. Such a process involves a η-spin flip which transforms a $+1/2$ holon onto a $-1/2$ holon and leads to the deviations $\Delta\eta = -1$ and $\Delta\eta_z = +1$. By use of the method of Ref. [10], we find $B^{(+1)}_{\vartheta}(k, \omega) = C^{(+1)}_{\vartheta,\,\iota'}(k)(\omega - E^{\iota'}_{ss}(k))^{\zeta^{\iota'}_{ss}(k)}$ for low positive values of $(\omega - E^{\iota'}_{ss}(k))$ where $C^{(+1)}_{\vartheta,\,\iota'}(k)$ is real and positive. The exponent $\zeta_{ss}(k) \equiv \zeta^{+1}_{ss}(k) = \zeta^{-1}_{ss}(-k)$ such that $\zeta_{ss}(k) = -1 + \sum_{\iota=\pm 1}\{\sqrt{K_\rho/2} + \Phi_{c0\,c1}(\iota\, 2k_F,\, q)\}^2 + \sum_{\iota=\pm 1}\{\Phi_{s1\,c1}(\iota\, k_F,\, q)\}^2$ refers to $k \in ([1-2\delta]\pi,\, \pi)$ and involves the phase shifts defined in Ref. [10] and the parameter $1/2 < K_\rho < 1$ [20]. The multiplicative constant $C^{(+1)}_{\vartheta,\,\iota'}(k)$ vanishes for $k = \iota'\pi$ and in the limits $\delta \to 0$ and $U/t_{\parallel} \to 0$ for all values of k. While $C^{(+1)}_{\vartheta,\iota'}(\iota'\pi) = 0$ for the oss and ess $c1$ pseudofermion singular branch lines, creation of a $-1/2$ Yang holon leads in the metallic phase to a single δ-function peak for $\vartheta = oss$, $B^{(+1)}_{oss,1}(\iota'\pi, \omega) = \delta N_a\, \delta(\omega - E_{uhb})$. This involves a rotation in η-spin space called *η-pairing mechanism* [19] such that $\Delta\eta = 0$ and $\Delta\eta_z = +1$. This peak has a significant fraction of the oss first UHB weight but does not contribute to the coupled-chain instability. It is absent for the ess function, such that $B^{(+1)}_{ess,1}(\iota'\pi, E_{uhb}) = 0$. Thus, for the latter function the whole first-UHB weight corresponds to the spectral feature generated by creation of the $c1$ pseudofermion. Consistently, although the oss and ess $c1$ pseudofermion singularities are controlled by the same exponents, $C^{(+1)}_{ess,\,\iota'}(k) > C^{(+1)}_{oss,\,\iota'}(k)$. Therefore, in our analysis of the coupled chains we do not consider the oss singularity.

The remaining more divergent one-chain singularities were already calculated in Ref. [4] and for small positive values of ω read $B^{(-1)}_{1p}(\mp k_F, -\omega) \propto B^{(+1)}_{1p}(\pm k_F, \omega) \propto (\omega)^{\zeta_{1p}}$ and $B^{(-1)}_{sdw}(\mp 2k_F, -\omega) \propto B^{(+1)}_{sdw}(\pm 2k_F, \omega) \propto (\omega)^{\zeta_{sdw}}$. The exponents are given by $\zeta_{sdw} = -[1 - K_\rho]$ and $\zeta_{1p} = -(3/2)\{1 - (1/3)[1/(2K_\rho) + K_\rho/2]\}$. Here $-1/2 < \zeta_{sdw} < 0$ and $-1 < \zeta_{1p} < -7/8$, whereas $\zeta_{ss}(k)$ is a monotonous decreasing function of k such that $\zeta_{ss}(k) > 0$ for $k = [1-2\delta]\pi$ and $\delta > 0$ and $\zeta_{ss}(k) \to -1$ as $k \to \pi$. For $U > 0$, ζ_{1p} and $\zeta_{ss}(k)$ are a decreasing and increasing function of δ, respectively, and $\zeta_{1p} = -7/8$ and $\zeta_{ss}(k) = -1$ for $\delta \to 0$. The regime of interest for our problem corresponds to small values of δ and $U/t_{\parallel}$ and of $[\pi - |k|] > 0$ for $\zeta_{ss}(k)$, when $E^{\iota'}_{ss}(k) \approx 2t_{\parallel}[\pi - |k|] + E_{uhb}$. The correlation pseudogap E_{uhb} is small for

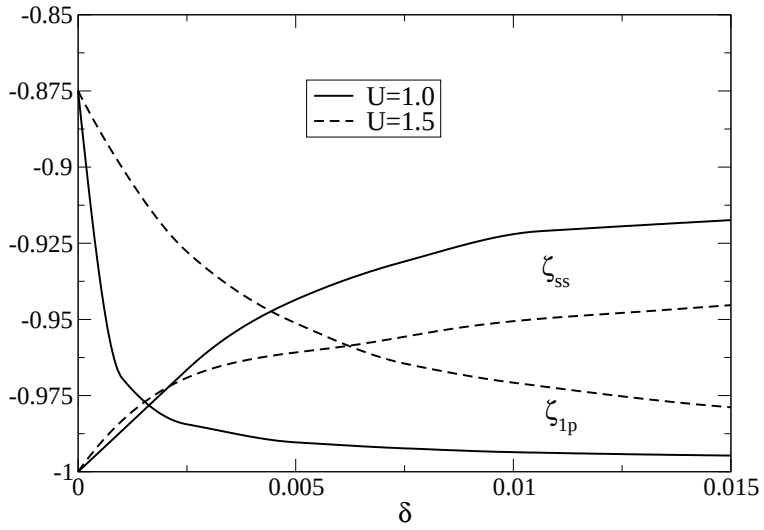

Fig. 2 – The exponents ζ_{1p} and $\zeta_{ss}([1-\delta/4]\pi)$ as a function of δ for $U = t_{\parallel}$ (full line) and $U = 1.5\,t_{\parallel}$ (dashed line).

$\delta \in (0,\, 0.02)$ and $U/t_{\parallel} \in (0,\, 2)$, when $E_{uhb} \approx \delta\, 2\pi t_{\parallel} + [8\sqrt{Ut_{\parallel}}/\pi]\, e^{-2\pi t_{\parallel}/U} << t_{\parallel}$ and the condition $E_{uhb} < t_{\perp} < t_{\parallel}$ is fulfilled. The exponents $\zeta_{ss}(k)$ for $k = (1-\delta/4)\pi$ and ζ_{1p} are plotted in Fig. 2 as a function of δ for $U/t_{\parallel} = 1.0$ and 1.5. (The exponent $\zeta_{sdw} > -1/2$ for $U > 0$ does not appear in the figure.) For low values δ below some value $\delta_0 = \delta_0(U/t_{\parallel})$ the exponent ζ_{ss} is the smallest and for $\delta > \delta_0$, ζ_{1p} is the smallest exponent. Let $k = (1-\delta/a)\pi$ with $a > 1$ in the argument of $\zeta_{ss}(k)$. For the same value of $U/t_{\parallel}$ (and a), δ_0 is an increasing function of a (and $U/t_{\parallel}$). Thus, the same qualitative results are reached for smaller values of $U/t_{\parallel}$ and δ_0, when the inequality $E_{uhb} << t_{\perp} << t_{\parallel}$ is fulfilled.

The renormalization of inter-chain couplings can be directly expressed in terms of the exponents which characterize the intra-chain spectral functions. For the 1D system the most divergent singularities occur in the $1p$ and ess functions and then we can restrict our attention to terms of the type,

$$\begin{aligned} \hat{H}_{Jc} &= \sum J\, f(k')f(k'')c^{\dagger}_{i,k-k',\uparrow}c^{\dagger}_{i,k',\downarrow}c_{i',k-k'',\uparrow}c_{i',k'',\downarrow}\,; \\ \hat{H}_{ph} &= \sum t_{\perp}c^{\dagger}_{i,k,\sigma}c_{i',k,\sigma}\,. \end{aligned} \tag{1}$$

Here $\hat{H}_{Jc}$ and $\hat{H}_{ph}$ refer to Josephson coupling and particle hopping, respectively, $f(k')$ defines the extended s pairing, and i and i' denote chain indices. These couplings are relevant and grow at a rate determined by the RG equations: $\partial\tilde{t}_{\perp}/\partial l = -\zeta_{1p}\,\tilde{t}_{\perp}$ and $\partial\tilde{J}/\partial l = -\zeta_{ss}\,\tilde{J} + \mathcal{C}\,\tilde{t}^{2}_{\perp}$. Here $l = -\log(\Lambda/\Lambda_0)$, Λ_0 is the initial value of the cutoff, $\tilde{t}_{\perp}$ and $\tilde{J}$ are the couplings divided by Λ, and $\mathcal{C}$ is a constant of order unity. Initially, the rate of growth of the Josephson coupling is proportional to $\tilde{t}^{2}_{\perp}$, and when this coupling reaches a value, in dimensionless units, of order $\tilde{t}^{2}_{\perp}$, its growth will be determined by the scaling dimensions. The analysis is valid until one of the couplings, $\tilde{t}_{\perp}$ or $\tilde{J}$, becomes of order unity, or when $\Lambda \sim E_{uhb}$. Integrating the RG flow, we obtain $\tilde{t}_{\perp}(l) = \tilde{t}_{\perp}(0)\, e^{|\zeta_{1p}|l}$ and $\tilde{J}(l) = \tilde{t}^{2}_{\perp}(0)\,[e^{2|\zeta_{1p}|l} - e^{|\zeta_{ss}|l}]/[2|\zeta_{1p}| - |\zeta_{ss}|] + \tilde{J}_0 e^{|\zeta_{ss}|l}$, where $\tilde{J}_0$ is the initial Josephson coupling. Note that as for spin singlet SC, all four-operator

Hamiltonian terms have couplings whose leading-order term is $\tilde{t}_\perp^2(0)\,[e^{2|\zeta_{1p}|l}]/[2|\zeta_{1p}|-|\zeta_\vartheta|]$. However, since $|\zeta_\vartheta|$ is largest for $\vartheta = ss$, the multiplicative constant $\tilde{t}_\perp^2(0)/[2|\zeta_{1p}|-|\zeta_\vartheta|]$ is also largest for $\vartheta = ss$. This is consistent with our choice of Hamiltonian terms, given in Eq. (1). As $2|\zeta_{1p}| > |\zeta_{ss}|$, the RG analysis breaks down at a scale $\log(\Lambda_{eff}/\Lambda_0) \approx -|\zeta_{1p}|^{-1}\log[\tilde{t}_\perp(0)]$. Provided that Λ_{eff} is greater than E_{uhb}, the analysis presented here gives a reasonable description of the properties of the system at energies higher than Λ_{eff}. Otherwise, the scaling equations cease to be valid at $\Lambda \sim E_{uhb}$. Let us assume that $\Lambda_{eff} \gg E_{uhb}$. At this scale both $\tilde{t}_\perp$ and $\tilde{J}$ become of order unity. Thus, the system presents two regimes: (i) at high energies or temperatures, greater than Λ_{eff}, the system is best described in terms of decoupled chains: the effective inter-chain coupling, in absolute units, decreases upon lowering the temperature or energy, as $t_\perp(T) \sim t_\perp(\Lambda_0)\,(T/\Lambda_0)^{1-|\zeta_{1p}|}$, inter-chain pair fluctuations increase, and the coherent contribution to the conductivity decreases; (ii) At sufficiently low energies or temperatures, as compared to Λ_{eff}, the leading terms in the effective Hamiltonian are a one-electron and a pair-hopping term of similar magnitude, $\sim \Lambda_{eff}$.

Note that the fact that the Josephson coupling has the largest scaling exponent ensures that its value at the point where the RG breaks down is the largest among all possible interchain interactions. The ground state is a strongly coupled superconductor which will show deviations from the BCS theory. The low-T phase can be viewed as realization of the inter-layer pair hopping model derived from coupled systems with strong RVB correlations [21]. Our analysis helps to delimit the region of validity of this hypothesis: (I) $t_\perp \ll t_\parallel$; (II) δ or $\bar{\delta}$ close to zero ; (III) E_{uhb} should be smaller than $t_\perp$. Note that the states which contribute to the pair-hopping instability change the η value, *i.e.* the initial GS and the excited states are not related by a mere rotation in η-spin space. Thus, the present pairing mechanism cannot be considered directly related to the processes analyzed in the SO(5) theory of the formation of d-wave SC in the cuprates [22]. The study of Ref. [12] also leads to singlet SC, but does not use the singularities of the uncoupled system. Our results are universal in the sense that from the RG scheme used here we can infer that the low-T phase of the coupled-chain system will show long-range SC order. However, the precise nature of this phase, and the symmetry of the order parameter is dependent on the arrangement of the chains within the material. Our results confirm that in 1D NFLs weak inter-chain hopping can induce SC and are consistent with the phase diagram of the series of quasi-1D compounds of Fig. 1. At low T these materials show a spin Peierls (SP) or SDW phase. Under pressure the $(TMTSF)_2X$ compounds are driven to a SC phase, which is removed again if one further increases the pressure. Consistently, for low values of δ we find that $E_{uhb} > t_\perp$ away from the small $U/t_\parallel$ region and according to the RG scheme of Ref. [4] the leading instability is towards a SDW state. Increasing pressure means here increasing $t_\parallel$ and $t_\perp$ without changing both U and the condition $t_\perp/t_\parallel << 1$. This drives the system into a small-$U/t_\parallel$ region, where E_{uhb} becomes smaller and $t_\perp$ is larger, and thus $E_{uhb} < t_\perp$, and the leading instability is towards a SC state. Since in the above $B_{ess}^{(+1)}(k,\omega)$ expression $C_{ess}^{(+1)}(k)$ decreases with decreasing $U/t_\parallel$ and vanishes as $U/t_\parallel \to 0$, further increase of pressure removes this state. In the absence of pair hopping, the renormalization of the inter-chain conductance ceases at Λ_{eff}, leading to an effective Fermi-liquid (FL) description at low T. The ability of our toy model to describe the diagram of Fig. 1 for $X = ClO_4$ can be further checked by deriving the values for its effective parameters from the NFL phase for $T \approx 10\,K$ and verifying whether they are consistent with the occurrence of the SC phase. The use of the PDT for the uncoupled system leads to a ω-dependent conductivity for the NFL phase including a small $\omega = 0$ Drude peak and a finite-energy absorption for $E_{opt} < \omega < E_{opt} + 8t_\parallel$ where $E_{opt} = E_{uhb} + \epsilon_{c1}^0(0)$. This absorption is generated by creation of a $-1/2$ holon and two c pseudofermion holes at q and $-q$ for $|q| \in (0,\, 2k_F \approx \pi)$. As found in Ref. [15] by use of

a preliminary version of the PDT, quantitative agreement with the $X = ClO_4$ and $T = 10\,K$ curve of Fig. 1(C) of Ref. [8] is obtained provided that $t_{\parallel} \approx 0.125\,eV$, $U/t_{\parallel} \approx 1.5$, and $\delta \approx 0.005$. Importantly, according to Fig. 2, these values are consistent with the occurrence of the SC phase for smaller T. Tight-binding model (TBM) calculations lead to $t_{\parallel} \approx 0.250\,eV$ [8] because the width of the c energy-band filled sea for $|q| < 2k_F \approx \pi$ is about twice that of the naive TBM band [7]. Our mechanism also explains the absence of SC phases in quasi-1D materials whose δ value is not close to commensurability. For TTF-TCNQ, $\delta = 0.41$ [2,3,10], the correlation pseudogap E_{uhb} is finite for all values of U, and thus the condition $E_{uhb} < t_{\perp}$ cannot be fulfilled. Although our analysis refers to $t_{\parallel} > t_{\perp}$, our results suggest that upon increasing $t_{\perp}/t_{\parallel}$ the SC instability will be extended to larger values of both δ and U. While for the quasi-1D system the instability corresponds to momentum values in the vicinity of π, we expect that for the high-T_c superconductors square lattice such an instability will appear in the vicinity of $[\pi, \pi]$ as $t_{\perp}/t_{\parallel} \rightarrow 1$.

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